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DETERMINATION OF LEVEL SENSITIVITY (FIELD CALIBRATION WITH THE LEVEL ON THE INSTRUMENT)

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A method is considered for determining level sensitivity without removing the level from the instrument. This has application for the hanging level and Horrebow-Talcott levels of the T-4 Wild Theodolite, the Zenith Wanschaff Telescope, and similar instruments.

By rotating the instrument in an inclined plane which is set in advance, the inclination angle varies as a function of the azimuths.

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ACKNOWLEDGMENT

This research note resulted from discussions between the author and the staff of the DMA Engineering School. The author undertook this research essentially outside of working hours to develop a new theory on this subject.

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ILLUSTRATIONS

DETERMINATION OF LEVEL SENSITIVITY
(FIELD CALIBRATION WITH THE LEVEL ON THE INSTRUMENT)

1. Introduction and Statement of the Problem. A method is considered for determining the level-vial sensitivity of the Zenith Wanschaff Telescope, the Wild T-4 Theodolite, and the plate level of other survey instruments. The level vials are calibrated without removing them from the instrument.

Sensitivity of the level vials varies with temperature, age, bubble length, stresses on the vial, the sense of bubble drift, and other factors. The calibration must be made at different ambient temperatures and the level sensitivity must be reduced to a standard temperature. The Wild T-4 hanging level is located at 90 degrees with respect to the optical axis of the telescope, while the Horrebow-Talcott level vials are parallel to it.

2. Theory of Level Sensitivity. In Fig. 1, let point O be the center of a sphere and OZ the direction of the plumb line. The plane through O, at right angles to OZ,

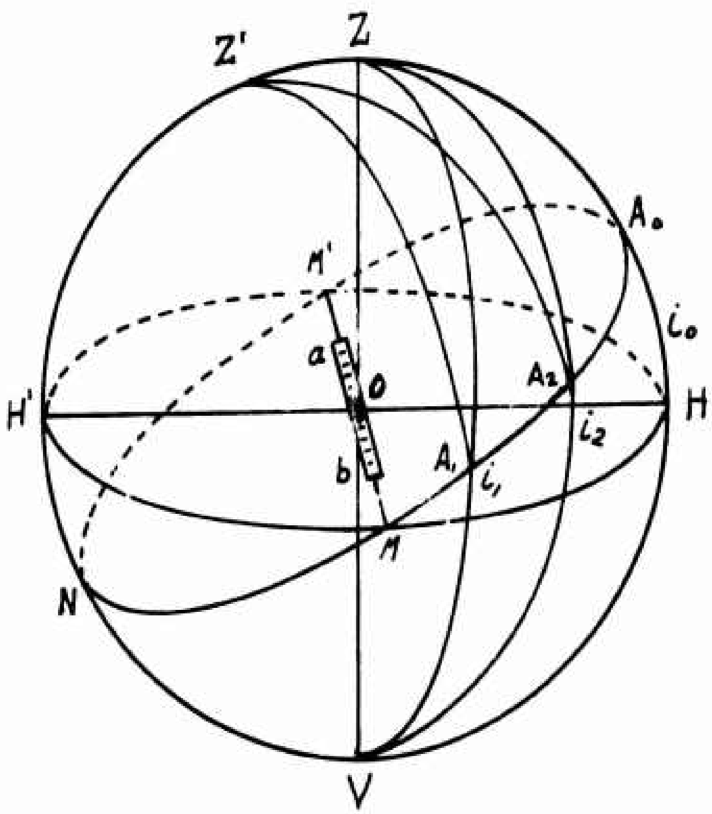

Fig. 1. Basic level-vial orientation.

is the plane of the horizon, cutting the sphere in the great circle H'MH. A plane through O, at right angles to OZ', cuts the sphere in the great circle NMA$_0$. Let i_0 be the inclination between these two planes. Let MM' be the line of intersection. Point Z is the pole of the great circle H'MH, and Z' is the pole of the great circle NMA$_0$. Hence M is at 90 degrees to both Z and Z'. Therefore, MA$_0$ = MH = 90°. Let a level vial be attached to the horizontal plane at O. The bubble position is referred to a scale on the tube itself.

After carefully leveling the base plate of the theodolite, represented by the plane H'MH, the vertical axis will have been brought into the vertical position; thus, when turning the theodolite about its vertical axis, the level bubble will remain stationary with respect to the central point of the level scale. The rotation is in the plane H'MH. On the other hand, when the base plate is inclined an angle i_0, the rotation of the theodolite will be in the plane NMA$_0$. The bubble of air will always stand at the highest point on the curve of the tube. Any change of the relative elevation of the two ends of the tube is accompanied by a corresponding change in the bubble position. Therefore, the bubble is moving to the end b when the rotation is in the sense MA$_1$A$_0$. The level position at the horizontal reading A$_1$ will indicate an inclination i_1, and inclination i_2 when the level occupies the position at the horizontal reading A$_2$. These inclinations are illustrated in Figs. 2 and 3. By moving the instrument in azimuth, we are varying the angle of inclination as a function of the azimuth readings. This allows us to define a method for determining the level sensitivity without removing the level vial from the instrument.

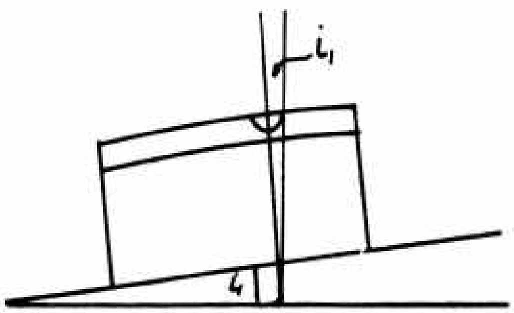

Fig. 2. Inclination i_1.

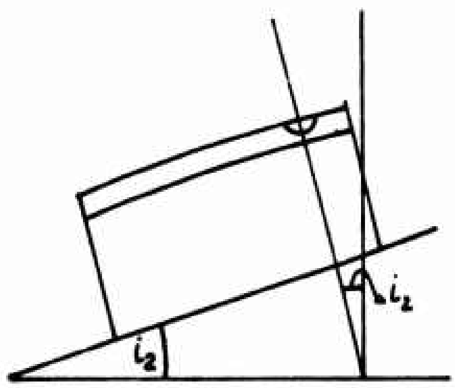

Fig. 3. Inclination i_2.

3. Trigonometrical Function Used for Determining the Level Sensitivity. The writer has developed new trigonometrical functions and analytical expressions, to be used in spatial triangulation, which have broad applications in fields of astronomy of position, star positioning, the determination of the motion of the poles, etc. One of these expressi ns shall be used here for the determination of the level sensitivity.

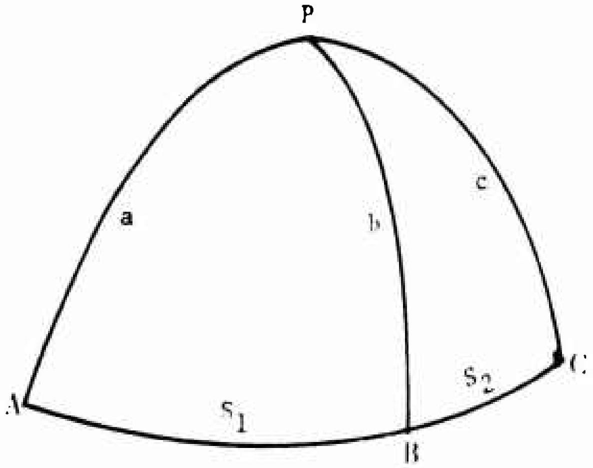

Fig. 4. Relationship between the five sides.

The relationship between five sides of adjacent spherical triangles may be established by this expression: Consider Fig. 4. Let S_1 and S_2 be a portion of the same great circle of arc. Let S_1 = AB and S_2 = BC. On the sphere, place an origin at P and let a, b, and c be the spherical coordinates of A, B, and C, respectively. An equation that relates the five sides is as follows:

$$\cos a \sin S_2 + \cos c \sin S_1 = \cos b \sin (S_1 + S_2). \qquad (1)$$

This shall be used for determining the level sensitivity.

4. **Determination of the Level Sensitivity.** The following relationships may be derived from Fig. 1:

$$a = ZA_1 = 90 - i_1$$
$$b = ZA_o = 90 - i_o$$
$$c = ZA_2 = 90 - i_2$$
$$S_1 = A_2 - A_1$$
$$S_2 = A_o - A_2$$

Therefore, the general equation (equation (1)) is set up as follows:

$$\sin i_1 \sin (A_o - A_2) + \sin i_o \sin (A_2 - A_1) = \sin i_2 \sin (A_o - A_1) \qquad (2)$$

The A's are the horizontal readings. Independent measurements can be made at will. Each one is made by changing the interval of azimuth by a constant angle of α minutes.

thus 10 independent measurements assume a total change in azimuth of 10α. Setting up the first measurement with an origin at A_1, the others are to be made at intervals $A_1 + \alpha$, $A_1 + 2\alpha$, $A_1 + 3\alpha$. . . . . . $A_1 + 10\alpha$. Let L_1, L_2, L_3. . . . . . L_{10} be central bubble-position readings with respect to the level-vial scale that correspond to the azimuths $A_1 + 0$, $A_1 + \alpha$, $A_1 + 2\alpha$, and so on.

By choosing measurements with large intervals, the influence of reading errors will be diminished when they are divided by a large number of intervals. Having this in mind, let

$$i = 1, 2, 3, 4, 5,$$

and

$$j = 6, 7, 8, 9, 10.$$

Then the general equation (equation (2)) becomes

$$\sin i_j \sin (A_o - A_i) - \sin i_i \sin (A_o - A_j) = \sin i_o \sin (A_j - A_i). \tag{3}$$

In the Wild T-4, the hanging level is located at 90 degrees with respect to the optical axis; therefore, ± 90 degrees must be added to the azimuth readings A_i and A_j. The azimuth that does not change is the reading that corresponds to the inclination i_o. Therefore

$$A_o - A_i \text{ must be } 90° - (A_o - A_i), \text{ and}$$

$$A_o - A_j \text{ must be } 90° - (A_o - A_j).$$

Thus, equation (3) becomes

$$\sin i_j \cos (A_o - A_i) - \sin i_i \cos (A_o - A_j) = \sin i_o \sin (A_j - A_i), \tag{4}$$

which is the equation to be applied for the Wild T-4 hanging level.

5. **Inclination Angles.** The inclinations i_i and i_j are small angles. Therefore, we can use the angles instead of the sines. On the other hand, the readings $(A_j - A_i)$ have a constant value. Also, $(A_o - A_i)$ and $(A_o - A_j)$ are small angles. We always hold them less than 1 degree, so it is justifiable to consider

$$\cos (A_o - A_i) \sim \cos (A_o - A_j) \sim 1.$$

Let ρ be the level sensitivity and L_i and L_j the scale vial readings. For the inclinations we have

$$i_i = \rho\,(L_i - L_m),\ \text{and}\ i_j = \rho\,(L_j - L_m), \tag{5}$$

where L_m is a constant (see paragraph 7). Let the constant difference in azimuth be defined in the following manner:

$$A_j - A_i = 5\,\alpha. \tag{6}$$

a. **Level Sensitivity.** Replacing the values given by equations (5) and (6) in equation (4), we can define the value of level sensitivity. The final formula for determining the level sensitivity is:

$$\rho = 206.265\ \frac{\sin i_o \sin 5\alpha}{L_j - L_i} \tag{7}$$

This formula gives the level sensitivity in seconds of arc. It is to be pointed out that the denominator $(L_j - L_i)$ has an almost constant value.

If the values of $\cos(A_o - A_i)$ and $\cos(A_o - A_j)$ are not close to 1, we must consider the following equation for determining the Wild T-4, hanging level sensitivity:

$$\rho = 206.265\ \frac{\sin i_o \sin (A_j - A_i)}{L_j \cos (A_o - A_j) - L_i \cos (A_o - A_i)} \tag{8}$$

To apply equation (7), the inclination i_o must be known. Before explaining how the inclination i_o is determined, we must first consider the bubble length and the drift interval (paragraphs 5b and 5c below).

b. **Bubble Length.** The level vial is partly filled with liquid, and when no inclination exists, a small bubble occupies the central position. The bubble length can be varied. It is chosen approximately between one-half and two-thirds of the level scale. If L_e and L_r are the end readings of the level-vial scale, the bubble length l_o is chosen between

$$\frac{1}{2}\ (L_r - L_e) < l_o < \frac{2}{3}\ (L_r - L_e). \tag{9}$$

c. **Bubble Drift.** Select an interval of five divisions of the level-vial scale for a variation in azimuth of a small angle α. Make a minimum of 10 independent observations in a Direct instrument position. After the last measurement, the azimuth has changed in 10 α with respect to the first reading. Then the instrument is turned 180 degrees in azimuth and a new set of 10 measurements is made by moving the telescope in reverse order – that is to say, by changing the azimuth intervals from 10α, 9α, 8α, ... α.

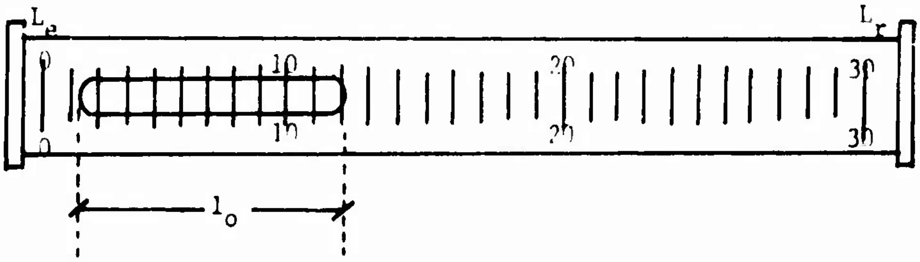

Fig. 5. Bubble length and level-vial scale.

In Fig. 5, let L_e and L_r be end-mark readings on the level-vial scale, and let N be the total number of intervals of the level-vial scale: then

$$N = L_r - L_e. \tag{10}$$

The bubble length occupies a space l_o of N. Let X be the free space that can be used for the total bubble drift; then

$$N = l_o + X, \tag{11}$$

where l_o is known. If the bubble length is chosen as

$$l_o = \frac{1}{2}\,(L_r - L_e), \tag{12}$$

we have

$$X_1 = \frac{1}{2}\,(L_r - L_e) = \frac{1}{2}\,N. \tag{13}$$

If instead we choose

$$l_o = \frac{2}{3}\,(L_r - L_e) \tag{14}$$

then

$$X_2 = \frac{1}{3}\,(L_r - L_e). \tag{15}$$

Let S be the bubble drift when the azimuth changes by an angle α. Let n be the total number of drifts (or repetitions), so that

$$n\,S = X$$

If we choose $n = 10$, we obtain

$$S = \frac{X}{10}. \tag{16}$$

For $x = x_1$, we have

$$S_1 = \frac{L_r - L_e}{20}. \tag{17}$$

and for $x = x_2$, we obtain

$$S_2 = \frac{L_r - L_e}{15}. \tag{18}$$

Having fixed the bubble drift S, S_1, or S_2, it remains to determine the inclination i_o.

6. Determination of the Inclination i_o. In Fig. 6, let A_m be the azimuth reading when the inclination is zero. We have chosen a bubble drift S for an azimuth change of angle α. Let us choose $\alpha = 5$ minutes; therefore, the 10 independent measurements require a total interval of $10\alpha = 50$ minutes, which in Fig. 6 is represented by the arc $A_m A_x$. We need to know, to a rough approximation, the reading A_m. The exact value can be determined from the observations. We use a value $A_m - A_x = 1$ degree, to be sure that the 10 independent observations can be used. The inclination i_x is

$$i_x = \rho''\, X = nS. \tag{19}$$

where X is chosen according to the bubble length selected, and ρ'' is the level sensitivity, of which an approximate value is supplied by the manufacturer. Applying equation (1), we have

$$\cos 90^\circ \sin (A_o - A_x) + \sin i_o \sin (A_x - A_m) = \sin i_x \sin (A_o - A_m). \tag{20}$$

Inasmuch as $(A_o - A_m)$ is very nearly 90 degrees, we can let

$$\sin\,(A_o - A_m) = 1.$$

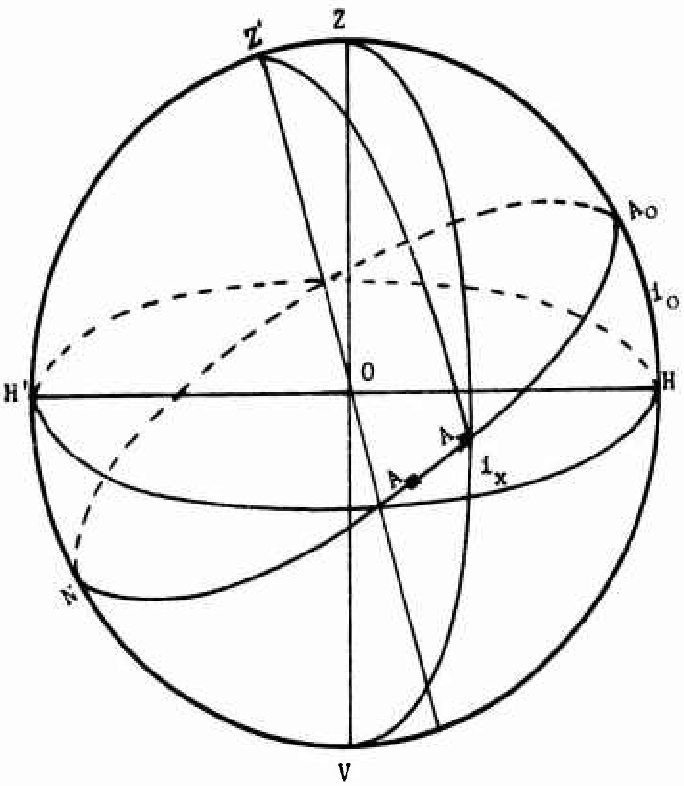

Fig. 6. Basis for determining inclinations and azimuths.

Therefore, the inclination i_o is computed from

$$\sin i_o = \sin i_x / \sin (A_x - A_m). \tag{21}$$

Reading A_m is the azimuth reading that corresponds to the level position showing virtually no variation in the bubble position. We fix $(A_x - A_m) = 1$ degree. Therefore, replacing i_x from equation (19), equation (21) becomes

$$\sin i_o = \frac{206{,}265\, \rho''\, X}{\sin 1^\circ}\ , \tag{22}$$

where

$$X = L_r - L_e - l_o .$$

The inclination i_o, as computed from equation (22), gives the maximum value. If we fix an inclination equal to one-half of the value computed through equation (22), we double the number of independent observations for determining the value of level sensitivity.

7. **Azimuth Direction for Fixing the Inclination.** Level the instrument carefully and place the telescope in the direction of one foot-screw. Let A_0 be the azimuth, and let Z_0 be the zenith distance of a reference field mark. Clamp the instrument. Fixing the telescope with a zenith distance $Z_0 + i_0$, reset the field mark by raising or lowering the instrument until the mark again appears in the central cross wires so that the inclination i_0 is fixed. When a field mark is not available in the direction which the telescope is fixed, we can proceed as follows: Place a mirror close to the telescope and turn the mirror in azimuth until any good field mark can be seen and used.

8. **Horrebow-Talcott Level Sensitivity.** The process of determining the Horrebow-Talcott level sensitivity is similar to the process used for the hanging level sensitivity. The only difference is that Horrebow-Talcott levels are placed in the vertical plane of the telescope. Therefore, there is no change in the horizontal readings. Consequently, for these levels we apply equation (3) in this form:

$$\sin i_j \sin (A_o - A_i) - \sin i_i \sin (A_o - A_j) = \sin i_o \sin (A_j - A_i). \qquad (23)$$

In order to make the inclination measurements i_i and i_j, the instrument must be rotated in azimuth about 90 degrees with respect to the reading A_o. The bubble motion must be checked to see that it moves in the right direction when the azimuth changes by an angle α.

The level sensitivity is determined by using the same equation (equation (7)), since we can set $\sin (A_o - A_i) \sim \sin (A_o - A_j) \sim 1$ for such small values of i_i and i_j. If $\sin (A_o - A_i)$ and $\sin (A_o - A_j)$ are not close to 1, we use the equation

$$\rho'' = 206{,}265 \; \frac{\sin i_o \sin (A_j - A_i)}{L_j \sin (A_o - A_i) - L_i \sin (A_o - A_j)} . \qquad (24)$$

9. **Determination of the Azimuth When the Inclination is Zero.** In order to determine the azimuth A_0 that would correspond with the bubble position showing no inclination, we proceed as follows.

In a rectangular-coordinates system, let the horizontal, axis-X, be the azimuth readings and the vertical, axis-y, be the inclination expressed in divisions of the level. Let $L_{o.1}$ be the central bubble readings and A_1 the azimuth when the instrument is clamped in a Direct Position. Let $L_{o.2}$ and $A_2 = A_1 + 180°$, the reading of the central bubble and azimuth when the instrument is in Inverse Position (by turning it 180 degrees). Then each pair of values of $L_{o.1}$ and $L_{o.2}$ will give a constant value for the mean,

$$L_m = \frac{1}{2}(L_{o.1} + L_{o.2}) = \text{constant.} \tag{25}$$

and the difference, $L_{o.1}-L_m$, will be the inclination, expressed in divisions of the level, for the azimuth A_1, A_2, and so on. Half of the inclinations

$$i = L_{o.1} - L_m \tag{26}$$

will be positive and the other half negative. If we plot each i of equation (26), we notice that if we join these points, they will lie on a straight line. The point where the straight line cuts the horizontal, axis-x, will be the azimuth A_o that corresponds to zero inclination.

Taking pairs of points whose inclinations are opposite in sign, we can compute the azimuth A_o as follows. Let these two points be

$$P_{o.1}\ [(L_{o.1} - L_m),\ A_1],\ \text{and}$$

$$P_{o.2}\ [(L_{o.2} - L_m),\ A_2].$$

Let h_1 represent the inclination of point $P_{o.1}$ and h_2 the inclination of point $P_{o.2}$. Then

$$h_1 = L_{o.1} - L_m,\ \text{and} \tag{27}$$

$$h_2 = L_{o.2} - L_m.$$

Let

$$x = A_o - A_1, \tag{28}$$

then from Fig. 7 it follows:

$$\frac{h_1}{x} = \frac{-h_2}{A_2 - A_1 - x}.$$

From this relationship

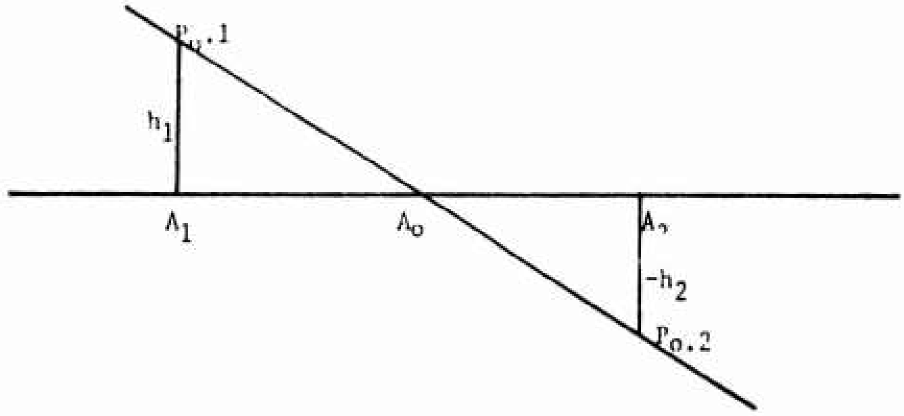

Fig. 7. Geometry for determining A_o.

$$x = \frac{h_1}{h_1 - h_2} (A_2 - A_1). \tag{29}$$

and from equation (28).

$$A_o = A_1 + x.$$

This is true for any other pair of points. Therefore, a general equation is set up:

$$A_o = A_i + x_i.$$

Factor x_i is computed from individual pairs through

$$x = \frac{h_1}{h_1 - h_2} (A_2 - A_1).$$

where h_1 and h_2 have the meaning expressed in equation (27). The negative sign in front of h_2 is required because in equation (27) $L_{o.2} < L_m$. We do not need to know A_o. In equation (8), the value for A_o to be used is the azimuth of the mark, because for this reading the inclination i_o is known. The value of L_m given by equation (25) is the one to be used in equation (5).

10. Discussion and Conclusions. The example used in this research note has the purpose of showing the process of computing the level sensitivity as it is described herein. Certainly, more calibrations must be conducted at different temperatures in order to have sufficient information for a complete analysis of the bubble-drift behavior and to see if the level sensitivity differs according to the sense of the bubble drift. This can be seen from the two values shown on Table 2 of the Appendix, where the two mean values have a difference of 0."034. This difference could be due to the bubble-drift motion. The analysis of bubble-drift behavior will give a clue as to whether or not the radius of curvature of different portions of the tube may be different. This defect will give different angular values to divisions of equal length in different portions of the tube.

To take into account the effect of temperature, let ρ_0 be the sensitivity value for an assumed temperature T_0, and let ρ be the value at temperature T. Then we have $\rho_0 = \rho + \rho (T - T_0) \beta$, in which ρ_0 and β are to be determined by least squares so as to satisfy the observed values of ρ at different temperatures. After ρ_0 and β are known, the sensitivity at any temperature T is

$$\rho = \frac{\rho_0}{1 + (T - T_0)\beta} .$$

The advantages in our method consist of the fact that the levels are not removed from the instrument itself for evaluation of the angular value of one division. Time is saved because this evaluation is made while the levels remain on the instrument. In addition, to determine the level calibration with a comparator by the standard method, the value of a division of the level results in partial revolution of the comparator's micrometer screw. The value of one revolution of this micrometer not only must be known accurately, but also its periodic and progressive errors must be accurately known. Also, the comparator may give an evaluation of the angular level division slightly different than when the evaluation is made with the level set up on the instrument. There is no guarantee of the level parallel position when it is set up in the comparator for the calibration, after the level is again set up on the instrument. Thus, the bubble displacement could not be on the same inclination, and this leads, therefore, to a different angular evaluation. Furthermore, our method allows us to calibrate the levels under the same conditions as those in which the instrument is used in field work. As can be seen by this new capability, the theodolite itself can be used as a comparator for calibrating and testing other levels or similar devices.

SAMPLE COMPUTATION

As an illustration of the method previously described, a practical application of the computational procedure is given. The instrument was first carefully leveled, after which the telescope was set up in the direction of one foot-screw. An auxiliary mirror was placed about 3 meters in front of the telescope. The mirror was turned until a good field mark was seen in the telescope. The mirror was then clamped. It is well to have the mark set as far away from the station as possible so that it will not be necessary to change the focus. Table 1 shows the observational data for the hanging level of the Wild T-4 Theodolite. The reference field mark was set as follows:

Azimuth mark:	351° 18′49″
Zenith distance:	255° 49′ 29.″9
Split level:	5.3 - 3.9

The telescope then was brought to the zenith distance $Z_2 = 254°59'29.''9$. Therefore, the inclination is $i_0 = Z_2 - Z_1 = 50'$. By activating the foot-screw, the telescope was moved up until the field mark was again on the same horizontal line, close to the cross wires. The first column gives the azimuth at which the instrument was clamped. The second column gives the bubble end readings. The capital letters, L and R, stand for left and right as the observer faces the level vial.

In Table 2, the first column shows the number of observations used for the Direct and Inverse instrument positions. The second and third columns were formed by substracting the azimuth mark $A_0 = 351°19'$ from each azimuth corresponding to the bubble-scale reading. The fourth column was formed from the mean bubble readings taken from the last column of Table 1. The fifth column is formed by dividing the constant $C = 21.''816$ by the number shown in column four. A_0-π was used for the Inverse Position. The sixth column shows the discrepancies between column five and the mean value. The resulting value for the level sensitivity is $\rho = 1.''047 \pm 0.''006$.

In Table 3, the first column indicates the setting used for the Direct and Inverse instrument positions. Corresponding azimuths are indicated in column two. The third column shows the mean value of the end-bubble readings taken from the last column of Table 1. The fourth column represents the mean value of column three. This is the value of L_m that appears in equation (5). The last column gives the inclination expressed in divisions of the level-vial scale. It is the term in parenthesis of equation (5), and is used in equation (25). These inclinations and the first azimuth

Table 1. Data for Wild T-4 Hanging Level

Azimuth ±90°	L	R	Length	L + R	½ (R + L)
351°00′	50.6	89.6	39.0	140.2	70.1
05	46.2	85.2	39.0	131.4	65.7
10	42.1	81.1	39.0	123.2	61.6
15	38.1	77.3	39.2	115.4	57.7
20	33.7	73.0	39.3	106.7	53.5
25	29.8	69.0	39.2	98.8	49.4
30	25.8	65.0	39.2	90.8	45.4
35	21.5	60.9	39.4	82.4	41.2
40	17.3	56.8	39.5	74.1	37.5
45	12.8	52.2	39.4	65.0	32.5
351 50	7.7	47.3	39.6	55.0	27.5
171°50′	51.0	90.8	39.8	141.8	70.9
45	47.1	86.8	39.7	133.9	66.95
40	43.0	82.6	39.6	125.6	62.80
35	38.5	78.2	39.7	116.7	58.35
30	34.2	74.0	39.8	108.2	54.10
25	30.1	69.9	39.8	100.0	50.0
20	26.0	65.8	39.8	91.8	45.9
15	21.9	61.6	39.7	83.5	41.85
10	17.6	57.3	39.7	74.9	37.45
5	13.1	53.0	39.9	66.1	33.05
171 00	8.0	48.0	40.0	56.0	28.00

NOTE:

Zenith Distance	Split Level	Azimuth's Mark
I 55° 49′ 29″9	5.3 – 3.9	351° 18′ 49″3
II 54° 59′ 29″9	5.3 – 3.9	
i_o 0° 50′ 0″0		

Table 2. Determination of Level Sensitivity

i - j	$A_i - A_o$	$A_j - A_o$	$L_j - L_i$	ρ	v
		Direct Position*			
1-6	90°19′	89°54′	20.7	1.″054	- 0.″010
2-7	14	49	20.3	1.075	+ .011
3-8	09	44	20.4	1.070	+ .006
4-9	90 04	39	20.2	1.080	+ .016
5-10	89 59	34	21.0	1.039	- .025
			Mean	1.064	
		Inverse Position*			
5-10	89°34	90°19	21.05	1.″037	+0.007
4-9	39	14	20.95	1.042	+ .012
3-8	44	09	20.90	1.044	+ .014
2-7	49	90 04	21.05	1.037	+ .007
1-6	89 54	89 59	22.00	0.992	- .038
			Mean	1.030	

* NOTE: Bubble motion was to the right in the Direct Position; to the left in Inverse Position.

Eq. (7) $$\rho = 206{,}265 \frac{\sin i_o \sin 5\alpha}{L_j - L_i} = \frac{C}{L_j - L_i}$$

Azimuth mark: $A_o = 351°19'$

Inclination: $i_o = 50'$

Azimuth difference: $5\alpha = 25'$

Constant: $C = 206{,}265 \sin i_o \sin 5\alpha$

$C = 21.''816$

$$\rho = \frac{21.''816}{L_j - L_i}$$

$\rho = 1.''047 \pm 0.''006$

Table 3. Determination of the Inclination

No.	Azimuth	$L_o = \frac{1}{2}(R + L)$	Mean	Inclination
1	351°00′	70.10	49.05	+21.05
	171 00	28.00		
2	351 05	65.70	49.38	+16.32
	171 05	33.05		
3	351 10	61.60	49.52	+12.08
	171 10	37.45		
4	351 15	57.70	49.72	+ 7.98
	171 15	41.75		
5	351 20	53.55	49.72	+ 3.83
	171 20	45.90		
6	351 25	49.40	49.70	- 0.30
	171 25	50.00		
7	351 30	45.40	49.75	- 4.35
	171 30	54.10		
8	351 35	41.20	49.78	- 8.58
	171 35	58.35		
9	351 40	37.05	49.92	-12.87
	171 40	62.80		
10	351 45	32.50	49.72	-17.22
	171 45	66.95		

that corresponds to each inclination were used for determining the azimuth A_o, which corresponds to zero inclination (Fig. 8).

Table 4 was formed from data of Table 3. The first column gives the setting used, the numbers corresponding to column one of Table 3. The second column of Table 4 gives the difference values that correspond to the number on the setting (Table 3, column 5). The third column gives the value of the level sensitivity computed in the same manner as for the explanation of Table 2. The last column shows the discrepancies between column three and the mean value $\rho = 1''.043$.

Table 4. Determination of the Sensitivity

Set	$L_j - L_i$	ρ	V
1-6	21.35	1".023	-0".020
2-7	20.67	1.055	+ .012
3-8	20.66	1.056	+ .013
4-9	20.85	1.046	+ .003
5-10	21.05	1.036	- .007

$\rho = 21''.816/(L_j - L_i)$

$\rho = 1''.043 \pm 0''.006$

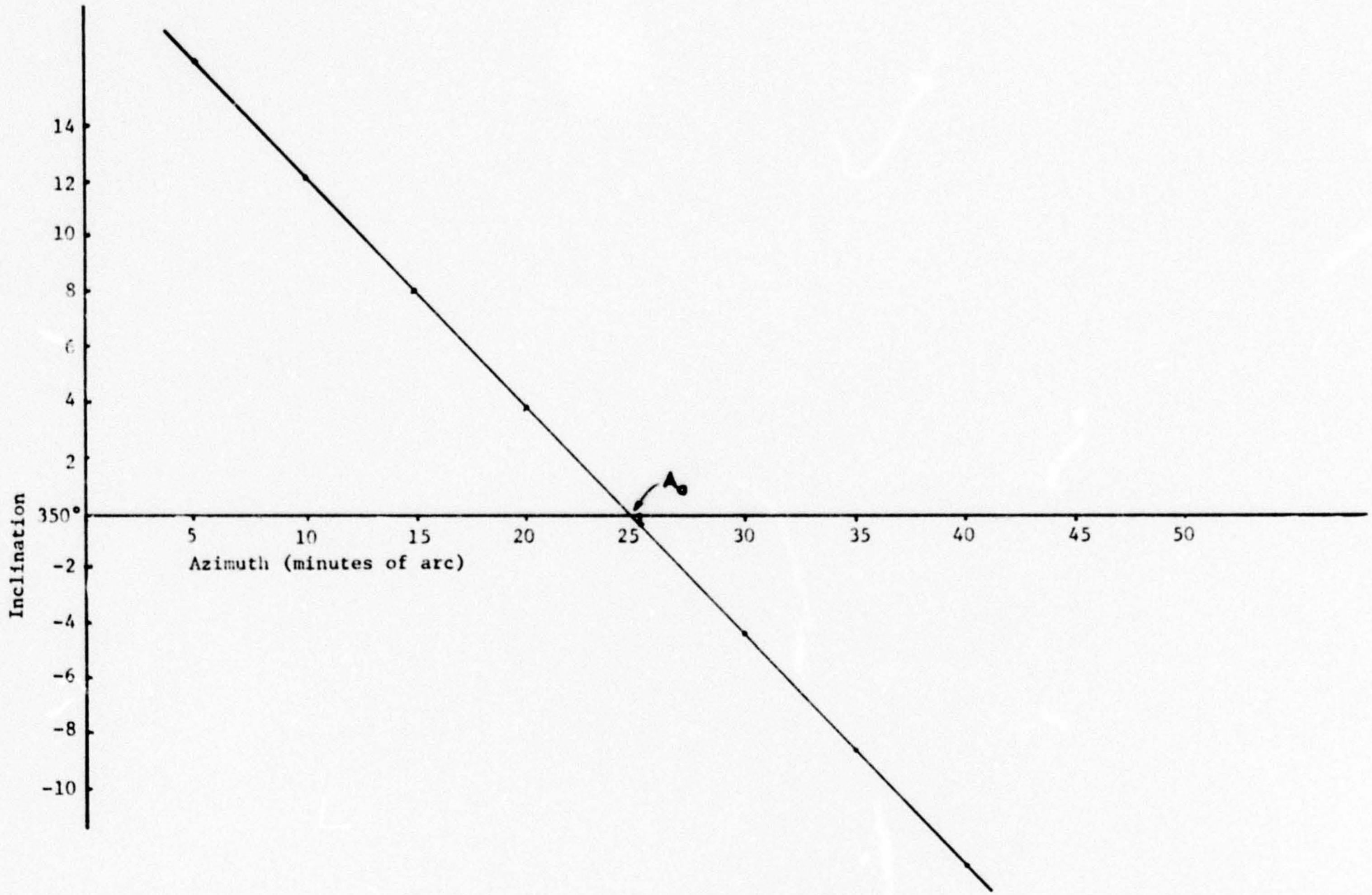

Fig. 8. Determination of the azimuth A_o corresponding to zero inclination.